MAGNETOSTATIC ENERGY DRAWDOWN IN A COSMIC PLASMA

by

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ABSTRACT

A charged particle in a cosmic plasma is often subjected to a static magnetic field and electromagnetic wave fields. These fields together produce a curved trajectory, and the particle feels a force due to this curvature. Analysis of the orbit shows that the particle takes energy from the magnetic field in the form of kinetic energy, and converts this energy to heat on collision with other particles. The result is a drawdown of the magnetostatic energy. Some cosmic environments where this effect may be relevant are suggested, and one of these (sunspot) is discussed in detail.

I. INTRODUCTION

An externally imposed static magnetic field in a steady state, linear plasma is considered inviolate with regard to local processes in the plasma. The attenuation of an electromagnetic (EM) wave in the plasma, for example, does not affect the magnetostatic energy. But this result overlooks certain electrodynamic processes which, when included, indicate a progressive drawdown of the said energy. Since the magnetic fields in most man-made plasmas are held constant by electromagnets, the effect here is inconsequential. In cosmic plasmas, however, the effect is relevant. The drawdown effect was first suggested in dielectric fluids (De, 1994) which share many properties of plasmas (De, 1988). Wave-magnetic field energy interchange in free space has also been explored (De, 1993).

II. CURVATURE OF A CHARGED PARTICLE TRAJECTORY

A charged particle (mass m, charge q) having a velocity $\mathbf{v}$ ($v \ll c$, the speed of light) in a static magnetic field $\mathbf{B}_o = B_o\, \mathbf{a}_z$, and the electric field $\mathbf{E} = E_o \sin \omega t\, \mathbf{a}_x$ of an EM wave of frequency f is considered ($\omega = 2\pi f$; $B_o \gg b$, the wave magnetic field). The rest frame of the problem is fixed to the source of $\mathbf{B}_o$. This geometry obtains in many cosmic environments. Initially, the thermal velocity of the particle is neglected. It then moves under the force

$$\mathbf{F} = q\, [\, \mathbf{E} + \mathbf{v} \mathrm{x} \mathbf{B}_o\,] \qquad (1)$$

along a loop or a series of loops ("Orbit 0") executed in one time-period. The elemental length traversed in time Δt on this path is found to be

$$\Delta \mathbf{S}_o(\mathbf{B_o}) = C_o\, [\omega_o \cos \omega_o t - \omega(\omega/\omega_o) \cos \omega t]\Delta t\, \mathbf{a}_x$$
$$- C_o\, [\omega_o \sin \omega_o t - \omega \sin \omega t]\Delta t\, \mathbf{a}_y \qquad (2)$$

where $\omega_o = qB_o/m$, and $C_o = (E_o/B_o)\omega_o^2/[\omega(\omega^2 - \omega_o^2)]$. There arises a corresponding orbital magnetic moment $\boldsymbol{\mu}_0$. The change in the kinetic energy of the particle during Δt is

$$\Delta W_o = \mathbf{F}.\Delta \mathbf{S}_o = q\mathbf{E}.\Delta \mathbf{S}_o \qquad (3)$$

III. SPIN AND THE CURVATURE FORCE

A particle in the above orbit turns about an axis (parallel to the z axis) translating with its center. The average angular frequency of this spin is ω. It arises due to coupling between the particle's intrinsic magnetic moment $\boldsymbol{\mu}_s$ and its orbital magnetic moment $\boldsymbol{\mu}_0$.

The spin in turn modifies the path of the particle. To see this, the charge of the particle is assumed to be uniformly and linearly distributed as a rigid body parallel to the instantaneous direction of motion of the center of the particle. This distribution has already been justified, e.g., in calculating the classical self-force on a radiating electron (Cf. Sommerfeld, 1964). The linear extent of the distribution is s_o (~ 6.10^{-15} m for electrons). The curvature of the trajectory causes a velocity differential along the rigid body in the rest frame [Figure 1(a)]. This velocity is $\mathbf{V}(\mathbf{s}')$, and $\mathbf{v} = \mathbf{V}(0)$. The rest frame magnetic field $\mathbf{b}_{curv}$ at $\mathbf{s}'$ is (*op. cit.*):

$$\mathbf{b}_{curv}(\mathbf{s}') = (\mu_o q/4\pi s_o)\int [\mathbf{V}(\mathbf{s}) \times \mathbf{d}/d^3 + \mathbf{d} \times \dot{\mathbf{V}}(\mathbf{s})/cd^2 - \{\mathbf{d} \times \mathbf{V}(\mathbf{s})\}\{\mathbf{d}.\dot{\mathbf{V}}(\mathbf{s})\}/c^2d^3]\, ds \qquad (4)$$

where $\mathbf{d} = \mathbf{s} - \mathbf{s}'$ (μ_o = free space permeability). For $\omega \ll c/s_o$ the acceleration terms above, and the induced electric field $\mathbf{e}_{curv}$ ($\nabla \mathbf{x} \mathbf{e}_{curv} \sim -\dot{\mathbf{b}}_{curv}$) may be ignored. Being coupled to $\mathbf{B}_o$, the particle can exert a self-force to move itself. This force per unit length on the element ds' is [Figure 1(b)]

$$\mathbf{f}_{curv}(\mathbf{s}') = (q/s_o)\, \mathbf{V}(\mathbf{s}') \times \mathbf{b}_{curv}(\mathbf{s}'), \qquad (5)$$

and can also be understood thus: Since $\mathbf{b}_{curv}$ opposes $\mathbf{B}_o$, there is a reduction of the net magnetic force at ds' by the amount f_{curv}. When viewed from the rest frame there is a longitudinal stress, and a net force $\mathbf{F}_c$ on the particle along the unit normal $\mathbf{a}_S$ to the trajectory:

$$\mathbf{F}_c = \int \mathbf{f}_{curv}(\mathbf{s}') \sin \angle(\mathbf{s}',\mathbf{f}_{curv})\, ds' = q\, \mathbf{v} \times \mathbf{b}_c \qquad (6)$$

This defines $\mathbf{b}_c$ as the effective curvature field, opposing $\mathbf{B}_o$. The field actually seen by the particle is $\mathbf{B} = \mathbf{B}_o + \mathbf{b}_c$. Equation (1) becomes

$$\mathbf{F} = q\, [\, \mathbf{E} + \mathbf{v} \mathbf{x} \mathbf{B}] \qquad (7)$$

IV. THE MODIFIED ORBIT

The orbit of Equation (7) can be developed as a perturbation on Orbit I [$\Delta \mathbf{S}_I = \Delta \mathbf{S}_o(\mathbf{B})$]. The magnetic force $\mathbf{F}_m = q\mathbf{v}\mathbf{x}\mathbf{B}$ is directed along $\mathbf{a}_S$. The balance of forces along $\mathbf{a}_S$ at any point O at time t is considered [(Figure 2(a)]. Orbit I requires that $\mathbf{F}_m.\Delta\mathbf{S}_I = 0$. Hence, over the next time interval Δt, the particle would proceed normal to $\mathbf{F}_m$. The centrifugal force $\mathbf{F}_R = (mv^2/R)\mathbf{a}_S$ must balance the component of $\mathbf{F}$ along $\mathbf{a}_S$ so that

$$\mathbf{F}_\perp = q(mv^2/qR - vB + E \sin \theta)\, \mathbf{a}_S \tag{8}$$

must be a null vector, where R is the radius of curvature of Orbit I at O. This defines a function $b_c(v)$. But a function $b_c(v)$ has already been specified by Equations (4)-(6). Therefore the new function is inadmissible, and the actual path $\Delta\mathbf{S}_{II}$ (Orbit II) differs from $\Delta\mathbf{S}_I$. The particle feels a force $\mathbf{F}_\perp \neq 0$ and executes a constrained motion along $\Delta\mathbf{S}_{II}$ [Figure 2(b)]:

$$\Delta\mathbf{S}_{II} = \Delta\mathbf{S}_I + (\Delta t^2/2m)\, \mathbf{F}_\perp. \tag{9}$$

V. TRANSFORMATION OF MAGNETOSTATIC ENERGY

The change in kinetic energy now is

$$\Delta W_{II} = \mathbf{F}.\Delta\mathbf{S}_{II} = q\mathbf{E}.\Delta\mathbf{S}_{II} + q[\mathbf{v}\mathrm{x}\mathbf{B}_o].\Delta\mathbf{S}_{II} + q[\mathbf{v}\mathrm{x}\mathbf{b}_c].\Delta\mathbf{S}_{II} \tag{10}$$

There are only two sources of energy in the medium: the EM wave, and the energy field $U_o = B_o^2/2\mu_o$. The first term on the right hand side above represents work done by the wave. The third term represents a reduction in this work, since $\mathbf{b}_c$ derives from the spin which in turn derives from the wave. In the second term the force is formed of the velocity and the magnetostatic field. But the particle cannot take energy from itself.

Therefore only the field U_o can be the source of the time-average kinetic energy:

$$\xi = \langle q[\mathbf{vxB}_o].\Delta S_{II}\rangle = (\mu_o^{½}q/\sqrt{2}m) \langle vF_{\perp}\Delta t^2\rangle U_o^{½} \qquad (11)$$

VI. DRAWDOWN

When the particle collides with another particle in a plasma ξ is converted to heat, raising the temperature of the plasma. Since after collision the subject particle has no memory of its direction, ξ is never returned to the field U_o. This local depletion of U_o must eventually be accounted for globally at the source of $\mathbf{B}_o$. When the thermal velocity v_{th} is introduced, the field $\mathbf{b}_c$ still arises in a frame moving with the guiding center of the particle, so that the preceding considerations still apply (even with $v_{th} \gg v$).

Hence the rate of loss of energy per unit volume is $\partial U_o/\partial t = \xi N_q \nu$ W m^{-3}. Here ν is the number of collisions the subject particle (with a number density N_q) has with other particles per second. The following assumptions are implicit: (i) the average volume N^{-3} per particle in the plasma (density N) $\gtrsim$ the orbital volume, $\sim C_o^3$; (ii) the mean time ν^{-1} between collisions is $\gtrsim$ the period of the wave, 1/f; and (iii) $f \gtrsim f_{pl} = (N_e e^2/\varepsilon_o m)^{½}$, the plasma frequency ($N_e$, e = electron number density, charge; ε_o = permittivity of free space).

VII. QUANTITATIVE ESTIMATIONS

Computation of the drawdown effect involves a multitude of considerations. Here order-of-magnitude estimates are made for only a part of the effect, and for electrons which are more important than ions. An approximate analytical value for b_c in Equation (6) can be found when Orbit 0 is *nearly* circular, having a radius $R_o \gg s_o$. The circularity approximation is valid near $\omega \sim \omega_o$. The resonance at $\omega = \omega_o$ is avoided. Now the value b_{co} of b_c is found to be

$$b_{co} \sim \mu_o ev\Lambda/4\pi s_o R_o \sim \mu_o e\Lambda/4\pi s_o,$$

where $v = R_o\omega$, and Λ is a factor of the order of unity arising from avoidance of singularities. The orbital magnetic moment of the electron is $\boldsymbol{\mu}_0 = -(e\omega R_o^2/2)\mathbf{a}_z$. Since $F_\perp$ is a continuous function near $b_c = 0$, and since near circularity of the orbit $v = R_o\omega$ and $\partial\theta/\partial b_c \approx 0$, it follows from Equation (8) that for $b_c \ll B_o$, $F_\perp \sim (\partial F_\perp/\partial b_c)b_{co} \sim evb_{co}$. Hence an estimate for ξ is

$$\xi_o \sim 2\pi^2 e^2 v^2 B_o b_{co}/m\omega^2$$

Setting $\omega \approx \omega_o$ and using $v = \pi e E_o/\sqrt{2}m\omega$ and the flux density $S = E_o^2/2\mu_o c$ of the wave, one has:

$$b_{co} \sim 0.5\, B_o$$

$$\xi_o \sim 3.10^{-26}\, B_o^{-2}\, S$$

Since the assumption $b_{co} \ll B_o$ turns out to be weak, the result is approximate on this ground as well. It shows reasonably however that a strong EM flux favors drawdown. This suggests some likely places where the effect may be relevant: the solar atmosphere ($B_o \sim 10^{-2}$-1 T; $f_o = \omega_o/2\pi$ in radiofrequency range) and possibly interior; magnetospheres of pulsars and neutron stars ($B_o \sim 10^8$ T; f_o in X-ray and gamma ray range); and the Earth's plasmasphere ($B_o \sim 10^{-5}$ T; f_o in radiofrequency range) illuminated by the quiet or active sun.

VIII. APPLICATION EXAMPLE: SUNSPOTS

In the sunspot umbra the magnetic field $\mathbf{B_o}$ is largely along the solar vertical, and the solar EM waves propagate also vertically. Hence the geometry of Section II holds for any polarization state of the waves. The physical conditions here may be described from observation and modeling (see e.g. Allen, 1983; Zirin, 1988; Semel *et al*, 1991; Skumanich, 1992). A density of neutrals $N_n \sim 10^{24}$ m^{-3}, and a temperature $T \sim 4.10^3$ K are assumed. With an ionization ratio of 10^{-6}, $N_e \sim 10^{18}$ m^{-3}. Then electron-neutral collisions dominate. For a typical $B_o \sim 0.4$T (4000 G), $f_o \sim 11$ GHz. The spectral flux density S_ω ($S = S_\omega \Delta f$, Δf being the bandwidth) of radio waves at the level of the photosphere can be estimated from its measured values at the Earth. The value of S_ω for the continuum emission at ~ 11 GHz here is about ~ 10^{-19} W m^{-2} Hz^{-1} (Jursa, 1985; Nelson et al, 1985). The corresponding photospheric value is ~ 5.10^{-15} W m^{-2} Hz^{-1}. Since the radio emission is broadband, one may use a bandwidth $\Delta f \sim 100$ MHz (< 1 per cent of f_o) and still use the relation $S = E_o^2/2\mu_o c$. For a weakly ionized plasma $\nu \sim 2.10^{-15} N_n T^{1/2}$ s^{-1} (Cf. Krall and Trivelpiece, 1973). Then one finds $\partial U_o/\partial t \sim -10^{-2}$ W m^{-3}. Generalizing the "local" results to global ones, one has the estimates

$$\partial B_o/\partial t \sim \mu_o(\partial U_o/\partial t)/B_o \sim -30 \text{ Gauss/day}$$

$$\partial T/\partial t \sim -2(\partial U_o/\partial t)/3kN_n \sim 40 \text{ K/day},$$

where k is the Boltzmann constant. These numbers indicate an evolution towards photospheric conditions over the life-time of a sunspot (many days).

IX. CONCLUDING REMARKS

An experimental verification of the drawdown effect may in principle be attempted in a discharge tube plasma in a waveguide. The field $\mathbf{B}_o$ is imposed by a permanent magnet. The parameters are so chosen as to predict a measurable weakening of the magnet over the duration of the experiment.

In sum, a phenomenon of potential interest in cosmic electrodynamics has been deduced and an example of its applicability demonstrated. In view of an earlier discussion (De, 1996), the drawdown process may represent rather a passage of mass to energy than a transformation of energy. Thus the present discussion may be of interest in the larger context of cosmology.

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FIGURE CAPTIONS

FIGURE 1. Illustration of the curvature fields and forces: (a) A charged particle moving in a curved trajectory is assigned a finite size, corresponding to a one-dimensional charge distribution tangential to the trajectory. The arrows indicate the velocities $\mathbf{V}(\mathbf{s})$ of different segments of this distribution. (b) The curvature forces $\mathbf{f}_{curv}$ on the different points $\mathbf{s}'$ of the charge distribution.

FIGURE 2. Illustration of the modified orbit: (a) Configuration of forces acting on the particle at any point O of the orbit. The centrifugal force is not shown. Path II is the actual direction of continuation of the orbit past O. Path I is the hypothetical direction of continuation. (b) The displacements $\Delta\mathbf{S}_I$ and $\Delta\mathbf{S}_{II}$.

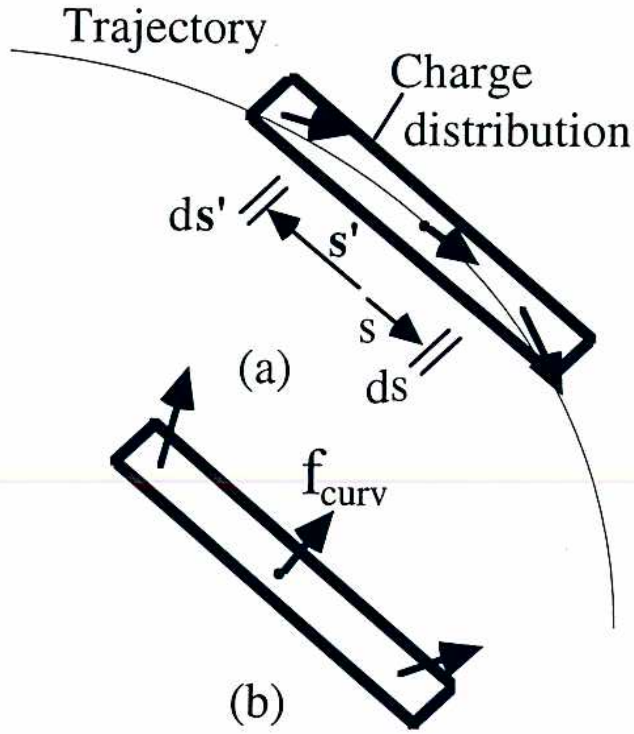

FIGURE 1

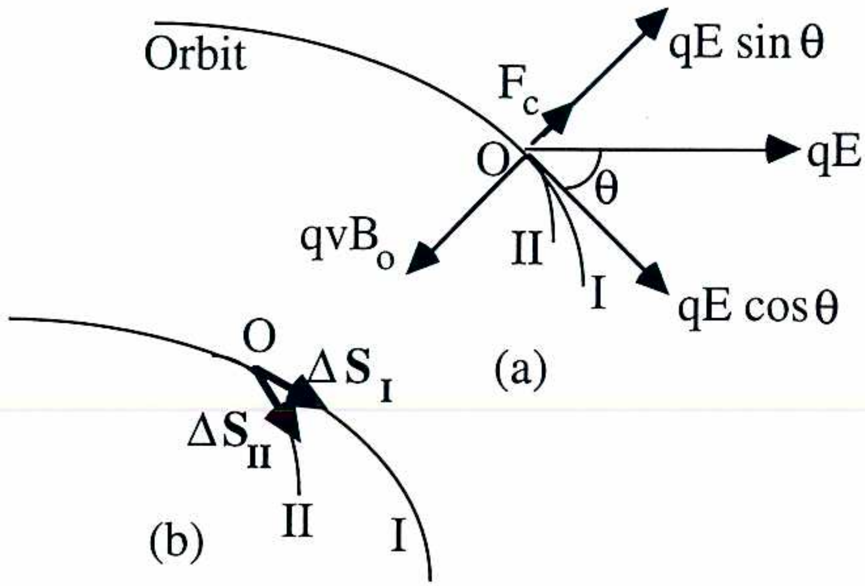
Orbit
F_c
$qE \sin\theta$
O
qE
θ
qvB_o
II
I
$qE \cos\theta$
O
$\Delta \mathbf{S}_{\mathbf{I}}$
(a)
$\Delta \mathbf{S}_{\mathbf{II}}$
(b)
II
I

FIGURE 2