LETTER TO THE EDITOR

Transformation of magnetostatic energy

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Abstract. Energy in a static magnetic field can be converted to other forms such as mechanical energy and heat. This is done by perturbing the magnetostatic energy by an electromagnetic wave—generating what has been described earlier as a 'companion wave'. When energy is drawn from this latter wave, the magnetic field decays.

The local energy in a static magnetic field in a region far away from the sources of this field is at present thought to be an abstract notion, since there are no known processes by which this energy can be locally altered or transformed. Straightforward consequences of an electromagnetic (EM) theory explored recently [1], however, point to the possibility that this energy may be perturbed in a certain way, and the perturbed energy can in turn be converted to other forms of energy. The present letter demonstrates this phenomenon by referring to a simple dielectric medium and using elementary physics.

A static, uniform magnetic field B_0 parallel to the z direction of a Cartesian coordinate system, initially in vacuum, is considered. A plane EM wave propagates in the x direction such that the electric and the magnetic fields of the wave are

$$e = e_0 \sin(\omega t - kx) a_y \tag{1}$$

$$b = b_0 \sin(\omega t - kx) a_z \tag{2}$$

where $\omega = 2\pi f$ (f = the frequency of the wave), $k = 2\pi/\lambda$ (λ = the wavelength), and e_0 and b_0 are the field amplitudes. The region of space under consideration is assumed to be so far away from the sources of the static magnetic field and the EM wave that these sources do not contribute to the energy conservation in local interactions as they occur. The magnetic energy densities in the static magnetic field and the wave magnetic field are $U_0 = B_0^2/2\mu_0$ and $u = b^2/2\mu_0$ (μ_0 = permability of free space), respectively. However, the net magnetic energy density in the medium is

$$U = U_0 + u + \hat{U} \tag{3}$$

with

$$u = u_0 \sin(\omega t - kx) \tag{4}$$

$$\hat{U} = \hat{U}_0 \sin(\omega t - kx) \tag{5}$$

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where $u_0 = b_0^2/2\mu_0$, $\hat{U}_0 = B_0 b_0/\mu_0$, and where $\hat{U} = B_0 b/\mu_0$ is the energy of interaction between the two magnetic fields, representing work done by one field on the other. It has been shown that the interaction energy $\hat{U}$ is just as real and as observable as the EM wave energy u [1]. The spatial and temporal oscillations of $\hat{U}$ constitutes a wave behaviour which has been termed a companion wave (since it accompanies an EM wave).

The fact that the flow of the interaction energy is as real and observable as that of the EM wave energy means that the former can perform work on and transfer energy to physical systems, as does the latter. To demonstrate this, a dielectric gas with a mass density ρ and a linear polarizability χ is chosen as the physical system. What results is a type of magnetohydrodynamic interaction that occurs in a purely dielectric fluid [2–5]. In the oscillatory electric field e the dipolar electric charges in each molecule alternately move towards and away from each other, and these movements of the charges in the magnetic fields b and B_0 give rise to microscopic forces on the molecule (for a discussion of this dielectric force, see, e.g., [6]). The net force on a unit volume is the cross-product of the dielectric polarization current density and the magnetic field:

$$F = (\omega\chi^{1/2}/c)u_0 \sin 2(\omega t - kx)a_x + (\omega\chi^{1/2}/c)\hat{U}_0 \cos(\omega t - kx)a_x \tag{6}$$

c being the velocity of light in the medium. The instantaneous velocity v of the gas is found from Newton's law $\mathrm{d}v/\mathrm{d}t = F/\rho$ to be

$$v = -(\chi^{1/2}/2\rho c)u_0 \cos 2(\omega t - kx)a_x + (\chi^{1/2}/\rho c)\hat{U}_0 \sin(\omega t - kx)a_x \tag{7}$$

so that the instantaneous kinetic energy is

$$\varepsilon = (\chi/8\rho c^2)u_0^2 \cos^2 2(\omega t - kx) + (\chi/2\rho c^2)\hat{U}_0^2 \sin^2(\omega t - kx) - (\chi/2\rho c^2)u_0\hat{U}_0 \cos 2(\omega t - kx)\sin(\omega t - kx) \tag{8}$$

and the time-average kinetic energy is

$$\langle\varepsilon\rangle = (\chi/16\rho c^2)u_0^2 + (\chi/4\rho c^2)\hat{U}_0^2. \tag{9}$$

The first term on the right-hand side represents an energizing of the dielectric by the EM wave, and is independent of the field B_0. The second term represents mechanical energy derived from the interaction energy. This, however, is not a primary energy source in the medium but is composed of the EM wave energy and the magnetostatic energy. Since the second term can be arbitrarily high for an arbitrarily high value of B_0 and may exceed the energy density of the EM wave, it follows that part of this mechanical energy must come from the energy in the field B_0. The second term is written separately as

$$\langle\varepsilon\rangle_i = C u_0 U_0 \tag{10}$$

with $C = \chi/\rho c^2$. This shows the relative contributions of u and U_0 to $\langle\varepsilon\rangle_i$. If U_0 is held constant and u_0 increases by Δu_0, then the portion of Δu_0 that goes to increase $\langle\varepsilon\rangle_i$ by $\Delta\langle\varepsilon\rangle_i$ is

$$\Delta\langle\varepsilon\rangle_i = C U_0 \Delta u_0.$$

Similarly, if u_0 remained unchanged and U_0 were higher by ΔU_0, the portion of ΔU_0 going to $\langle\varepsilon\rangle_i$ is

$$\Delta\langle\varepsilon\rangle_i = C u_0 \Delta U_0$$

If a part of the energy $\langle\varepsilon\rangle$ were to continually dissipate as heat, so would a part of $\langle\varepsilon\rangle_i$, and thus the energies u_0 and U_0 would both decrease. The result is an attenuation of the EM wave and a decay of the static magnetic field. This local loss of energy in the static magnetic field will eventually be communicated to the sources of B_0, which will act to complete the overall energy budget.

To complete the demonstration, it is noted from equations (9) and (10) that a loss of energy $\mathrm{d}\langle\varepsilon\rangle$ by dissipation may be written as

$$\mathrm{d}\langle\varepsilon\rangle = C_1 u_0\,\mathrm{d}u_0 + C U_0\,\mathrm{d}u_0 + C u_0\,\mathrm{d}U_0 \tag{11}$$

where $C_1 = C/8$. If the magnetic field B_0 *does not decay* (i.e., if $\mathrm{d}U_0 = 0$), then

$$\mathrm{d}u_0 = \mathrm{d}\langle\varepsilon\rangle/(C_1 u_0 + C U_0).$$

According to this, the higher the field B_0, the less the energy the wave has to provide to account for the same dissipation. Thus the dissipation may even exceed the available energy in the wave. Since this is not possible, it follows that some energy must be drawn from U_0, so that B_0 must decay.

The above discussion may now be rationalized by considering the actual mechanism by which a portion of U_0 couples into the motion of a molecule. The polarization charges in the molecule moving in the field B_0 execute a microscopic current loop which continually exchanges energy with this field. When the kinetic energy of the molecule dissipates as heat, so does a portion of the magnetic energy U_0. An example of a dielectric medium where this effect may be appreciable is the 'Rydberg gas' mode of excited atoms or molecules that achieve large polarizability just prior to ionization or dissociation [7, 8].

References

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If a part of the energy (a) were to continually dissipate as heat, so would a part of (c), and thus the energies w_{em} and U_0 would both decrease. The result is an attenuation of the EM wave and a decay of the static magnetic field. This latter loss of energy in the static magnetic field will eventually be communicated to the sources of B_0, which will act to complete the overall energy budget.

To complete the above discussion, it is found from equations (9) and (10) that a loss of energy $d(\eta)$ by dissipation may be written as

$$d(\eta) = C_1 \omega\, d\omega_0 + C'' \omega\, d\omega_0 + C_2\, dU_0 \qquad (11)$$

where $C_1 = C/\beta$. If the magnetic field B_0 does not decay (i.e., if $dU_0 = 0$), then

$$d\omega_0 = d(\eta)/(C_1\omega_0 + C' U_0).$$

According to this, the higher the field B_0, the less the energy the wave has to provide to account for the same dissipation. Thus the dissipation may even exceed the available energy in the wave. Since this is not possible, it follows that static energy must be drawn from U_0, so that B_0 must decay.

The above discussion may now be rationalized by considering the actual mechanism by which a portion of U_0 couples into the motion of a molecule. The polarization charges in the molecule moving in the field B_0 execute a macroscopic current loop which continually exchanges energy with this field. When the kinetic energy of the molecule dissipates as heat, so does a portion of the magnetic energy U_0. An example of a dielectric medium where this effect may be appreciable is the 'Rydberg gas' made of excited atoms or molecules that achieve large polarizability just prior to ionization or dissociation [7, 8].

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